ww^R only NPDA is possible

NPDA ⊃ DPDA

NPDA is more powerful than DPDA

DFA ≅ NFA
NPDA ⊃ DPDA

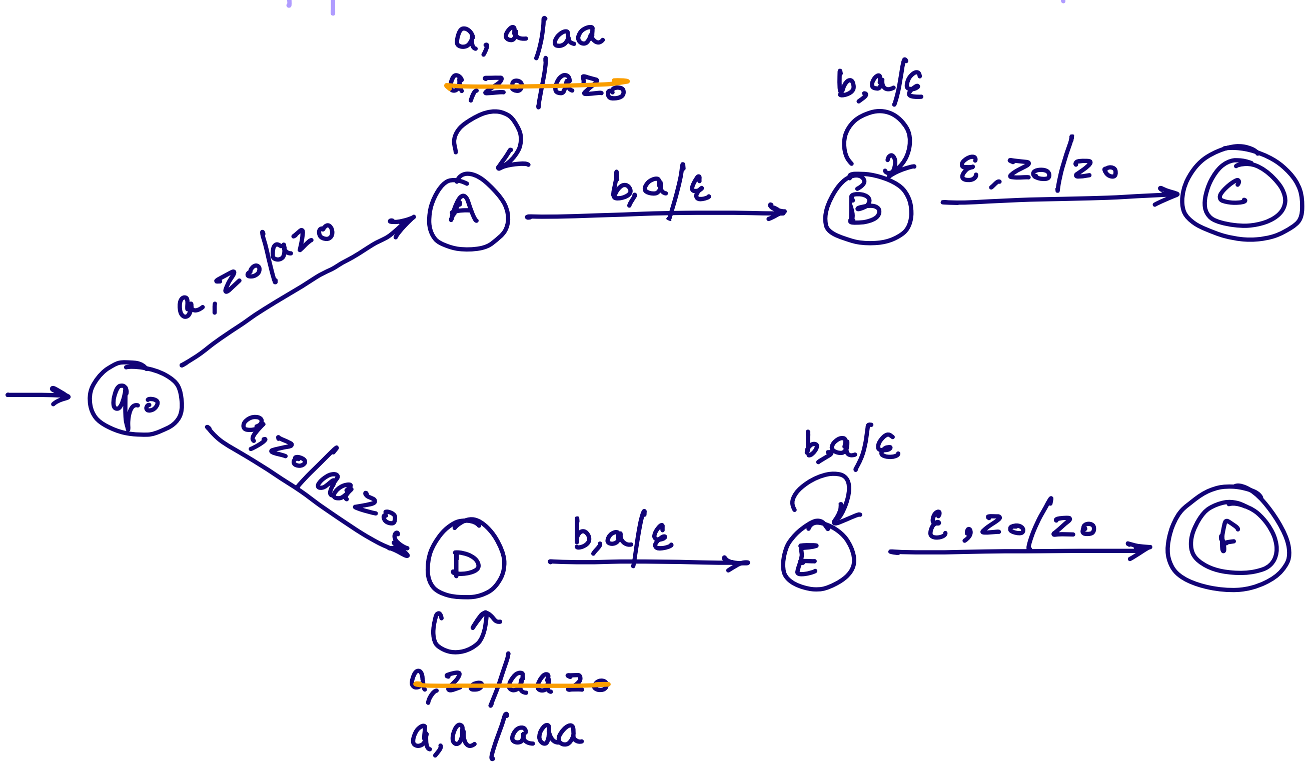
Eg: $L = \{a^n b^n \mid n \geq 1\} \cup \{a^n b^{2n} \mid n \geq 1\}$

$\{a^n b^n \mid n \geq 1\}$ → a: push a, b: pop a

$\{a^n b^{2n} \mid n \geq 1\}$ → a: push 2a's, b: pop 1a

aabbbb

NPDA



$(q_0, aabb, z_0)$

(A, abb, az_0) → (A, bb, aaz_0) →

(D, abb, aaz_0) → $(D, bb, aaaaz_0)$ →

(B, b, az_0)

$\downarrow$

(B, ε, z_0)

$\downarrow$

C

String accepted

$(E, b, aaaz_0)$

$\downarrow$

(E, ε, aaz_0)

(Dead Configuration)

Eg: $\{a^i b^j c^k d^l \mid i = k \text{ or } j = l\}$

no. of a's = no of c's

OR

no. of b's = no. of d's

Rewrite: $\{a^m b^j c^m d^l\} \cup \{a^i b^m c^k d^m\}$

$\{a^m b^j c^m d^l\}$:

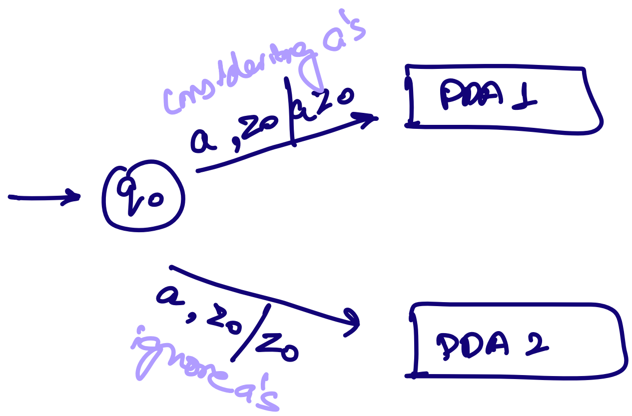
- Push a's
- ignore b's
- for c, pop a
- ignore d

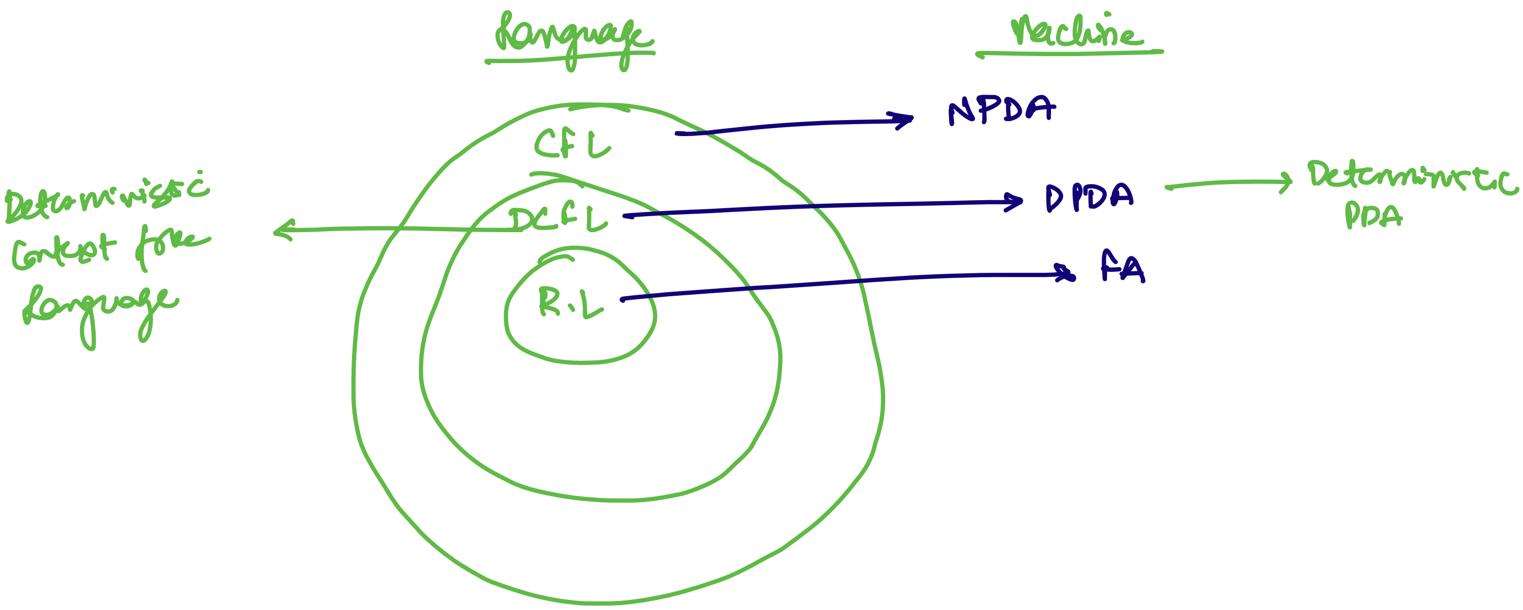
PDA1

$\{a^i b^m c^k d^m\}$:

- ignore a's
- push b's
- ignore c's
- for d, pop b

PDA2





Eg: $a^{m+n} b^{n} c^{m} \mid n, m \geq 1$

Regular ✗
DCFL ✓
CFL ✓

Eg: $a^{m} b^{m+n} c^{n} \mid n, m \geq 1$

RL ✗
DCFL ✓
CFL ✓

Eg: $a^{m} b^{n} c^{m+n} \mid n, m \geq 1$ RL ✗ DCFL ✓ CFL ✓

Eg: $a^{m} b^{m} c^{n} d^{n} \mid m, n \geq 1$ RL ✗ DCFL ✓ CFL ✓

push pop push pop

Eg: $a^{m} b^{n} c^{m} d^{n} \mid m, n \geq 1$ RL ✗ DCFL ✗ CFL ✗

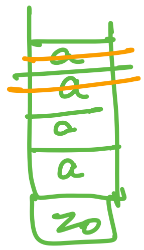
Eg: $a^{m} b^{n} c^{n} d^{m} \mid m, n \geq 1$ RL ✗ DCFL ✓ CFL ✓

Eg: $a^{m} b^{n} \mid m > n$

more a's than b's

RL ✗ DCFL ✓ CFL ✓

aaaa bb ε



ε , a → final state
(input alphabet: ε; stack top: a)

Eg: $a^n b^{2n} \mid n \geq 1$

Regular ✗
DCFL ✓
CFL ✓

Eg: $a^n b^{n^2} \mid n \geq 1$

3 b's 1 a pop

n = 3 $a^3 b^9$ → aaa bbb bbb bbb

n = 4 $a^4 b^{16}$ → aaaa bbbb bbbb bbbb bbbb

4 b's 1 a pop

RL ✗
DCFL ✗
CFL ✗

loop length will keep on varying

Eg: $a^n b^{2^n} \mid n \geq 1$

R ✗
DCFL ✗
CFL ✗

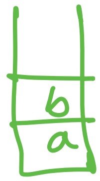
Eg: $ww^R \mid w \in (a,b)^*$

Regular ✗
DCFL ✗

CFL ✓

Eg: $ww \mid w \in (a,b)^*$

Eg: abab
$\underline{w}\ \underline{w}$



R ✗
DCFL ✗
CFL ✗

Eg: $a^n b^n c^m \mid n > m$

a^n → push, b^n → pop

R ✗
DCFL ✗
CFL ✗

Eg: $a^n b^n c^n d^n \mid n \le 10^{10}$ → finite language

abcd
aabbccdd
aaabbbcccddd
⋮
10^{10}

Regular ✓
DCFL ✓
CFL ✓

Eg: $a^n b^{2n} c^{3n} \mid n \ge 1$

1a : 5a's
1b : pop a
1c : pop a

$a^2 b^2 c^8$

aaaaa aaaaa
bottom top

RL ✗
DCFL ✗
CFL ✗

Eg: $xcy \mid x, y \in (0,1)^*$

→ 0's 1's c 0's 1's

c as a substring

Regular ✓
DCFL ✓
CFL ✓

Eg: $xx^R \mid x \in (a,b)^*, \ |x| = l$

a/b a/b a/b a/b — — . . . . a/b

$2 \cdot 2 \cdot 2 \cdots 2 = 2^l$

length e finite

RL ✓ CFL ✓
DCFL ✓

Eg: $ww w^R \mid w \in (a,b)^*$

CFL ✗

R ✗
DCFL ✗
CFL ✗

Eg: $a^n b^{3^n} \mid n \geq 1$

$n=1$ $a b^3$

$n=2$ $a^2 b^9$

$n=3$ $a^3 b^{27}$

⋮

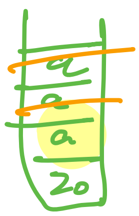
R ✗
DCFL ✗
CFL ✗

Eg: $a^m b^n \mid m \neq n$

R ✗
DCFL ✓
CFL ✓

$m > n$

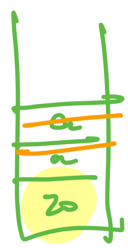
aaa bb ε



ε, a

$m < n$

aabbbε



b z0

Eg: $a^m b^n \mid m = 2n+1$

R ✗
DCFL ✓
CFL ✓

$a^{2n+1} b^n$

$a \cdot a^{2n} b^n$ → for 1 b: pop 1a

a → ignore

a^{2n} → for 2a's push 1a

Eg: $a^i b^{2j} \mid i \neq 2j+1$

$i > 2j+1$

$i < 2j+1$

a's are more | b's are more

Condition: | ε, a | b, 2₀

RX
DCFL ✓
CFL ✓

Eg: $a^{2^n} \mid n \geq 1$

eg: $a^{n!} \mid n \geq 1$

Eg: $a^m \mid m$ is prime

} RL ✗ DCFL ✗ CFL ✗

Eg: $a^k \mid k$ is even

$(a^0, a^2, a^4, a^6 \ldots)$

RL ✓
DCFL ✓
CFL ✓

Eg: $a^i b^j c^k \mid i > j > k$

PDA can't handle 3 comp.

RX
DCFL ✗
CFL ✗

Eg: $a^i b^j c^k \mid j = i + k$

$a^i b^i b^k c^k$

RX
DCFL ✓
CFL ✓

Eg: $a^i b^j c^k d^l \mid i=k$ or $j=l$

R ✗

DCFL ✗

CFL ✓

NPDA ✓

Eg: $a^i b^j c^k d^l \mid i=k$ and $j=l$

$a^m b^n c^m d^n$

R ✗

DCFL ✗

CFL ✗

Eg: $a^m b^l c^k d^n \mid m, l, k, n \geq 1$

↳ $a^+ b^+ c^+ d^+$

$aa^* bb^* cc^* dd^*$

R ✓

DCFL ✓

CFL ✓

Eg: $a^n b^{4m} \mid n, m \geq 1$

$aa^* (bbbb)(bbbb)^*$

R ✓

DCFL ✓

CFL ✓

Eg: $a^{2n+1} \mid n \geq 1$

2n+1: odd no

FA is possible

R ✓

DCFL ✓

CFL ✓

Eg: $a^{n^n}, n \geq 1$

R ✗

n^n

DCFL ✗
CFL ✗

Eg: $w \mid w \in (a,b)^*$, $|w| \geq 100$

$\underbrace{(a+b)(a+b) \text{———} (a+b)}_{100 \text{ times}} (a+b)^*$

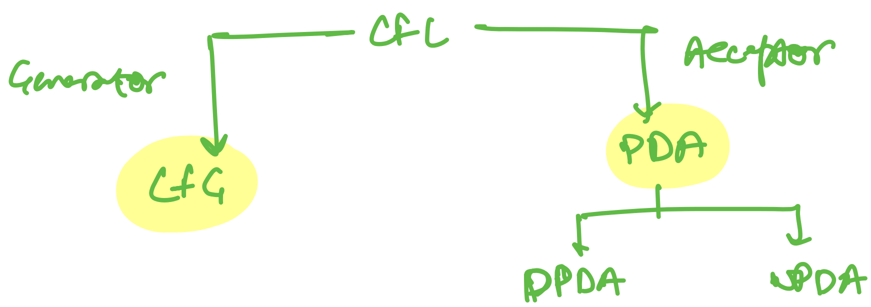
R ✓
DCFL ✓
CFL ✓

Eg: $w \mid w \in (a,b,c)^*$, $n_a(w) = n_b(w) = n_c(w)$

→ 3 cmp ✗

R ✗
DCFL ✗
CFL ✗

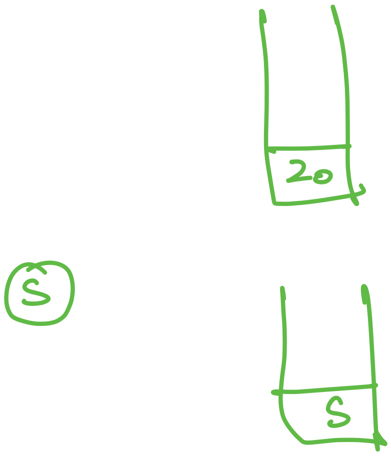
Equivalence of PDA & CFG



CFG to PDA

1. Convert CFG productions to GNF
2. PDA will have 1 state {q}
3. Start symbol of CFG will be initial symbol in PDA

$A \rightarrow \underline{a}$ VVV
$S \rightarrow \epsilon$
$A \rightarrow a$



4. Non terminal symbol (variable):

$A \rightarrow xYZ$

$\delta(q, \varepsilon, A) \rightarrow (q, xYZ)$

5. for each terminal:

'x'

$\delta(q, x, x) \rightarrow (q, \varepsilon)$

Q:

$S \rightarrow aAA$

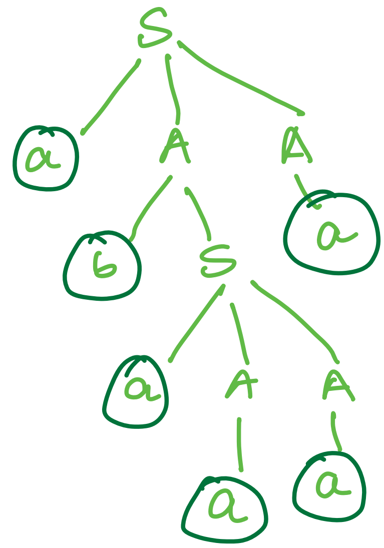
$A \rightarrow aS \mid bS \mid a$

1. GNF ✓
2. $\{q\}$
3. S
4. $\delta(q, \varepsilon, S) \rightarrow (q, aAA)$

   $\delta(q, \varepsilon, A) \rightarrow (q, aS) \mid (q, bS) \mid (q, a)$

5. $\delta(q, a, a) \rightarrow (q, \varepsilon)$

$\delta(q, b, b) \rightarrow (q, \varepsilon)$



string

abaaaa

↓ PDA accept?

state — input — bottom

$\delta(q, abaaaa, S)$

↓

$\delta(q, \underline{a}baaaa, \underline{a}AA)$

↓

$\delta(q, baaaa, AA)$

↓

$\delta(q, \underline{b}aaaa, \underline{b}SA)$

↓

$\delta(q, aaaa, SA)$

↓

$\delta(q, \underline{a}aaa, \underline{a}AAA)$

↓

$\delta(q, aaa, AAA)$

↓

$\delta(q, \underline{a}aa, \underline{a}AA)$

|

$\delta(q, aa, AA)$

↓

$\delta(q, aa, aA)$

↓

$\delta(q, a, A)$

↓

$\delta(q, a, a)$

$\delta(q, \epsilon)$ **Accepted**

PDA → CFG

PDA = $Q, \Sigma, \delta, q_0, Z_0, F, \Gamma$

- Q → Set of States
- Σ → input alphabets
- δ → Transition
- q_0 → initial State
- Z_0 → bottom of stack
- F → final States
- Γ → Stack Alphabets

Grammar:

NT → $S \cup [q, A, P]$ (triplet)

$q, P \in Q$

$A \in \Gamma$

1. $S \rightarrow [q_0, Z_0, P]$ for each P

2. **PDA:** $\delta(q, x, A) = (P, B_1 B_2 \ldots B_m)$

(state, input, stack)

production

Grammar: $[q, A, q_{m+1}] \rightarrow x [P, B_1, q_2][q_2, B_2, q_3] \ldots [q_m, B_m, q_{m+1}]$

3. $\delta(q, x, A) = (P, \varepsilon)$

$[q, A, P] \rightarrow x$

$x \in \Sigma \cup \{\varepsilon\}$

- Σ → input alphabet
- $\cup$ → union
- ε → epsilon

Example:

$\{ \underset{Q}{\{q_0, q_1\}}, \underset{\Sigma}{\{a, b\}}, \underset{\delta}{\delta}, \underset{q_0}{q_0}, \underset{Z_0}{Z_0}, \underset{F}{\phi}, \underset{\Gamma}{\{Z_0, x\}} \}$

PDA Define

PDA
- acceptance by final state
- acc. by empty stack

acceptance by final state ≈ acc. by empty stack

✓ $\delta(q_0, a, Z_0) = (q_0, xZ_0)$

✓ $\delta(q_0, a, x) = (q_0, xx)$

✓ $\delta(q_0, b, x) = (q_1, \varepsilon)$

$\delta(q_1, b, x) = (q_1, \varepsilon)$

$\delta(q_1, \varepsilon, Z_0) = (q_1, \varepsilon)$

$a^n b^n \mid n \geq 1$

1. $S \rightarrow [q_0, z_0, P]$ for each P, $P \in Q$

$Q : \{q_0, q_1\}$

$P \rightarrow q_0$

$\rightarrow q_1$

$S \rightarrow [q_0, z_0, q_0]$

$S \rightarrow [q_0, z_0, q_1]$

2. $\delta(q, x, A) = (P, B_1 B_2 \cdots B_m)$

production

$[q, A, q_{m+1}] \rightarrow x\,[P, B_1, q_2][q_2, B_2, q_3] \cdots [q_m, B_m, q_{m+1}]$

$q \quad x \quad A \qquad P \quad B_1 \; B_2$

$\delta(q_0, a, z_0) = (q_0, x z_0)$

$[q_0, z_0, \_\,] = a\,[q_0, x, \_\,][\,\_\,, z_0, \_\,]$

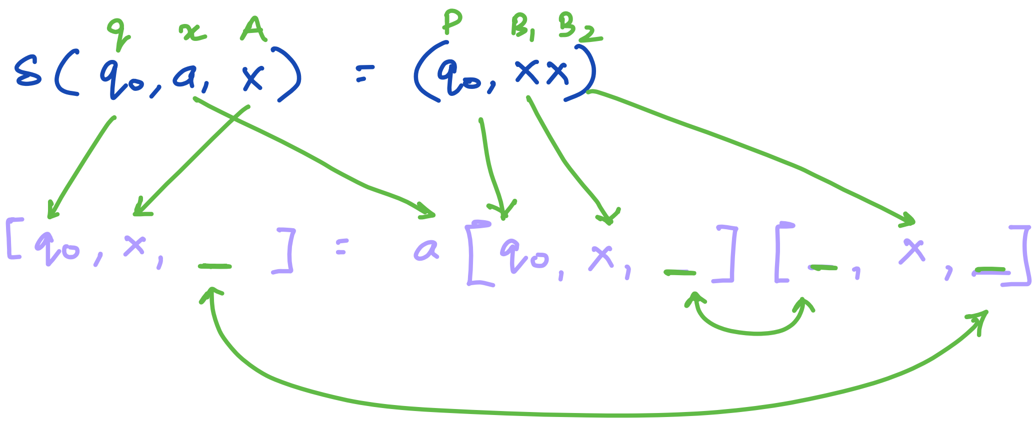
all possible combinations

$[q_0, z_0, q_0] = a\,[q_0, x, q_0]\,[q_0, z_0, q_0]$

$[q_0, z_0, q_0] = a\,[q_0, x, q_1]\,[q_1, z_0, q_0]$

$[q_0, z_0, q_1] = a\,[q_0, x, q_0]\,[q_0, z_0, q_1]$

$[q_0, z_0, q_1] = a\,[q_0, x, q_1]\,[q_1, z_0, q_1]$



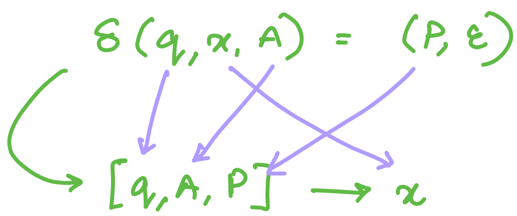
$[q_0, x, q_0] = a\,[q_0, x, q_0]\,[q_0, x, q_0]$

$[q_0, x, q_0] = a\,[q_0, x, q_1]\,[q_1, x, q_0]$

$[q_0, x, q_1] = a\,[q_0, x, q_0]\,[q_0, x, q_1]$

$[q_0, x, q_1] = a\,[q_0, x, q_1]\,[q_1, x, q_1]$

3.



$\delta(q_0, b, x) = (q_1, \varepsilon) \longrightarrow [q_0, x, q_1] \rightarrow b$

$\delta(q_1, b, x) = (q_1, \varepsilon) \longrightarrow [q_1, x, q_1] \rightarrow b$

$\delta(q_1, \varepsilon, z_0) = (q_1, \varepsilon) \longrightarrow [q_1, z_0, q_1] \rightarrow \varepsilon$

$S \rightarrow [q_0, z_0, q_0]$

$S \rightarrow [q_0, z_0, q_1]$

$[q_0, z_0, q_0] = a\,[q_0, x, q_0]\,[q_0, z_0, q_0]$

$[q_0, z_0, q_0] = a\,[q_0, x, q_1]\,[q_1, z_0, q_0]$

$[q_0, z_0, q_1] = a\,[q_0, x, q_0]\,[q_0, z_0, q_1]$

$[q_0, z_0, q_1] = a\,[q_0, x, q_1]\,[q_1, z_0, q_1]$

$[q_0, x, q_0] = a\,[q_0, x, q_0]\,[q_0, x, q_0]$

$[q_0, x, q_0] = a\,[q_0, x, q_1]\,[q_1, x, q_0]$

$[q_0, x, q_1] = a\,[q_0, x, q_0]\,[q_0, x, q_1]$

$[q_0, x, q_1] = a\,[q_0, x, q_1]\,[q_1, x, q_1]$

$[q_0, x, q_1] \rightarrow b$

$[q_1, x, q_1] \rightarrow b$

$[q_1, z_0, q_1] \rightarrow \varepsilon$

Remove useless symbols

Triplet which is present on RHS of production but not present in LHS.

$[q_1, z_0, q_0]$

$S \rightarrow [q_0, Z_0, q_0]$

$S \rightarrow [q_0, Z_0, q_1]$

$[q_0, Z_0, q_0] = a\,[q_0, x, q_0]\,[q_0, Z_0, q_0]$

~~$[q_0, Z_0, q_0] = a\,[q_0, x, q_1]\,[q_1, Z_0, q_0]$~~

$[q_0, Z_0, q_1] = a\,[q_0, x, q_0]\,[q_0, Z_0, q_1]$

$[q_0, Z_0, q_1] = a\,[q_0, x, q_1]\,[q_1, Z_0, q_1]$

$[q_0, x, q_0] = a\,[q_0, x, q_0]\,[q_0, x, q_0]$

$[q_0, x, q_0] = a\,[q_0, x, q_1]\,[q_1, x, q_0]$

$[q_0, x, q_1] = a\,[q_0, x, q_0]\,[q_0, x, q_1]$

$[q_0, x, q_1] = a\,[q_0, x, q_1]\,[q_1, x, q_1]$

$[q_0, x, q_1] \rightarrow b$

$[q_1, x, q_1] \rightarrow b$

$[q_1, Z_0, q_1] \rightarrow \varepsilon$

$[q_1, x, q_0]$

$S \rightarrow [q_0, Z_0, q_0]$

$S \rightarrow [q_0, Z_0, q_1]$

$[q_0, Z_0, q_0] = a\,[q_0, x, q_0]\,[q_0, Z_0, q_0]$

~~$[q_0, Z_0, q_0] = a\,[q_0, x, q_1]\,[q_1, Z_0, q_0]$~~

$[q_0, Z_0, q_1] = a\,[q_0, x, q_0]\,[q_0, Z_0, q_1]$

$[q_0, Z_0, q_1] = a\,[q_0, x, q_1]\,[q_1, Z_0, q_1]$

$[q_0, x, q_0] = a\,[q_0, x, q_0]\,[q_0, x, q_0]$

~~$[q_0, x, q_0] = a\,[q_0, x, q_1]\,[q_1, x, q_0]$~~

$[q_0, x, q_1] = a\,[q_0, x, q_0]\,[q_0, x, q_1]$

$[q_0, x, q_1] = a\,[q_0, x, q_1]\,[q_1, x, q_1]$

$[q_0, x, q_1] \rightarrow b$

$[q_1, x, q_1] \rightarrow b$

$[q_1, Z_0, q_1] \rightarrow \varepsilon$

(q_0, x, q_0)

$A \to aAA$

$\to aaAA\ A$

$S \to [q_0, z_0, q_0]$

$S \to [q_0, z_0, q_1]$

~~$[q_0, z_0, q_0] = a [q_0, x, q_0] [q_0, z_0, q_0]$~~

~~$[q_0, z_0, q_0] = a [q_0, x, q_1] [q_1, z_0, q_0]$~~ ✓

~~$[q_0, z_0, q_1] = a [q_0, x, q_0] [q_0, z_0, q_1]$~~

$[q_0, z_0, q_1] = a [q_0, x, q_1] [q_1, z_0, q_1]$

~~$[q_0, x, q_0] = a [q_0, x, q_0] [q_0, x, q_0]$~~ ✓ (A = a A A)

~~$[q_0, x, q_0] = a [q_0, x, q_1] [q_1, x, q_0]$~~ ✓

~~$[q_0, x, q_1] = a [q_0, x, q_0] [q_0, x, q_1]$~~

$[q_0, x, q_1] = a [q_0, x, q_1] [q_1, x, q_1]$

$[q_0, x, q_1] \to b$

$[q_1, x, q_1] \to b$

$[q_1, z_0, q_1] \to \varepsilon$

(q_0, z_0, q_0)

~~$S \to [q_0, z_0, q_0]$~~ }

$S \to [q_0, z_0, q_1]$

~~$[q_0, z_0, q_0] = a [q_0, x, q_0] [q_0, z_0, q_0]$~~

~~$[q_0, z_0, q_0] = a [q_0, x, q_1] [q_1, z_0, q_0]$~~

~~$[q_0, z_0, q_1] = a [q_0, x, q_0] [q_0, z_0, q_1]$~~

$[q_0, z_0, q_1] = a [q_0, x, q_1] [q_1, z_0, q_1]$

~~$[q_0, x, q_0] = a [q_0, x, q_0] [q_0, x, q_0]$~~

~~$[q_0, x, q_0] = a [q_0, x, q_1] [q_1, x, q_0]$~~

~~$[q_0, x, q_1] = a [q_0, x, q_0] [q_0, x, q_1]$~~

$[q_0, x, q_1] = a [q_0, x, q_1] [q_1, x, q_1]$

$[q_0, x, q_1] \to b$

$[q_1, x, q_1] \to b$

$[q_1, z_0, q_1] \to \varepsilon$

Final Productions:

$S \rightarrow [q_0, z_0, q_1]$ — A

A: $[q_0, z_0, q_1] = a\,[q_0, x, q_1]\,[q_1, z_0, q_1]$ — B, C

B: $[q_0, x, q_1] = a\,[q_0, x, q_1]\,[q_1, x, q_1]$ — B, D

B: $[q_0, x, q_1] \rightarrow b$

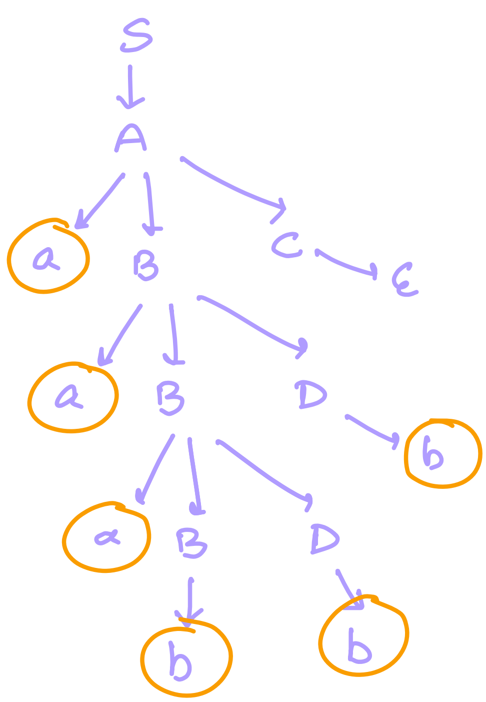
D: $[q_1, x, q_1] \rightarrow b$

C: $[q_1, z_0, q_1] \rightarrow \varepsilon$

↓

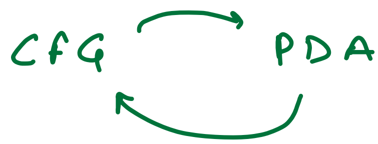
$S \rightarrow A$
$A \rightarrow aBC$
$B \rightarrow aBD$
$B \rightarrow b$
$D \rightarrow b$
$C \rightarrow \varepsilon$

} Context free Grammar



$a^n b^n \mid n \geq 1$

$a^3 b^3$



CFG & PDA both are equivalent in power.